Interaction Between Backfill and Rock Mass in Narrow Stopes

Interacción Entre el Relleno y la Masa de Roca en Excavaciones Escalonadas Estrechas

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Abstract
The design of backfilled stopes requires that the interaction that develops at the interface between the fill material and surrounding rock mass be evaluated. In this paper, the authors present analytical and numerical results from calculations performed to estimate the mechanical response of backfilled openings, emphasizing the effect of load transfer along the interface between the rock and relatively soft fill. The results indicate that the arching effect that can develop in the backfill may have a significant influence on load distribution along the walls and at the bottom of narrow stopes. The discussion that follows the comparison between sample numerical and analytical calculations, highlights some of the key influence factors. It also emphasizes the limitations of commonly available tools, which do not represent adequately key components of the interaction between fill and stope walls.

Resumen
El diseño de excavaciones escalonadas rellenas requiere la evaluación de la interacción en la interfaz entre el material de relleno y la masa de roca que lo rodea. En este artículo, los autores presentan resultados analíticos y numéricos de cálculos realizados para estimar la respuesta mecánica de aberturas rellenadas, acentuando el efecto de la transferencia de carga a lo largo de la interfaz entre la roca y un relleno relativamente blando. Los resultados indican que el efecto arco que puede desarrollarse en el relleno podría tener una influencia significativa en la distribución de la carga a lo largo de las paredes y en el fondo de excavaciones escalonadas estrechas. La discusión que sigue después de la comparación entre cálculos numéricos y analíticos, destaca algunos de los factores clave influyentes. Igualmente, acentúa las limitaciones de las herramientas comúnmente disponibles, las cuales no representan adecuadamente componentes clave de la interacción entre el relleno y las paredes de la excavación escalonada.

1 INTRODUCTION
Over the last twenty-five years or so, underground mines have increasingly relied on backfilling for ground support in stopes. With this augmented use of backfill came a need to better understand the mechanical behavior of the different types of fill materials, which generally include a significant proportion of residues from the mining and milling operations. In fact, the environmental benefits of returning part of the tailings and rock wastes underground has largely contributed to the rapid acceptance of backfilling as a valuable management option, which has significant advantages from a ground control perspective and from a mine wastes management point of view.

Different types of mine backfill have been described by Hassani and Archibald (1998), who also provide typical values for their mechanical properties. Other data on the mechanical behavior of backfill can be found in a number of recent publications, including: Benzaazoua et al. (2002), Belem et al. (2000, 2002). The available data indicate that backfill, even when it includes a significant proportion of binder, remains a relatively soft material compared to the surrounding rock mass. Because of that, the interaction forces that develop between the rock

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and fill may induce some form of load transfer along the interfaces, which in turn favors arching in the backfill. Such load transfer needs to be understood to properly assess the behavior of backfilled stopes, and optimize their design.

Knutsson (1981) suggested that the pressure generated on the walls of backfilled stopes could be evaluated using calculation methods developed for the storage of particulate materials in deep containment structures such as silos and bins (see also Hustrulid et al. 1989). Such methods are typically based on arching theories (even though Knutsson did not refer explicitly to arching), which take into account friction between the wall and fill, and the ensuing load distribution (e.g., Bowles 1988). This arching phenomenon is briefly described below, and a specific analytical solution (somewhat similar to the one employed by Knutsson 1981) is used to calculate the loads on adjacent walls and floor in vertical narrow openings. These analytical results are then compared to numerical calculations. It is shown that even though arching can also be observed in numerical simulations, the nature and magnitude of the load distribution is generally quite different from those based on arching theories. At this point, some important limitations remain with analytical and numerical calculation methods commonly available, as both approaches neglect some key aspects.

2 ARCHING PHENOMENON

The interaction forces (and pressure) between backfill and surrounding rock mass is sometimes evaluated using the ground reaction (response) curves (Fig. 136 in Hoek and Brown 1980). This approach is regularly used to establish the relationship, as a force-displacement diagram, between ground support loading and rock mass convergence (Einstein and Schwartz 1979; Hoek et al. 1995).

However, in the case of relatively narrow backfilled stopes, this approach neglects any form of arching due to the frictional load transfer that can develop along the walls when a material with a different stiffness is placed into an opening. Arching is a well known process, and it has been introduced in different ways to analyze a variety of situations, including wall pressure in silos (Jaky 1948; Richards 1966; Blight 1986), vertical stress and support requirements above tunnels and other underground situations (Terzaghi 1943; Ladanyi and Hoyaux 1969; Iglesia et al. 1999), earth pressure on retaining walls in trenches (Spangler and Handy 1984; Take and Valsangkar 2001), and weight on conduits in ditches (Spangler 1962; McCarthy 1988). When arching develops in a particulate medium, a large part of the weight of the material located above the main "arch" position can be transferred to the abutments, thus reducing the load magnitude to the walls and floor below the arch. A number of conditions need to be satisfied for this phenomenon to occur. Among these, the geometry of the opening must be relatively narrow, and frictional forces must develop between the two adjacent materials having different stress-strain responses.

The validity of arching concepts and of existing theoretical treatments has been demonstrated experimentally (at least partially) for some specific cases by Knutsson (1981), Blight (1986), Frydman and Keissar (1987), Jarrett et al. (1995), and Take and Valsangkar (2001).

When applied to openings such as backfilled stopes, arching theories, attributed to Janssen and a few others (Terzaghi 1943; McCarthy 1988; Iglesia et al. 1999; Take and Valsangkar 2001), indicate that there can be a significant vertical stress reduction in the lower part of the stope compared to the overburden weight. This load transfer is due to frictional (and cohesive in some cases) interaction between the fill and surrounding walls. Because of that, pressure exerted on the floor and walls may be quite different from values deduced from classical pressure theories developed for exposed earth retaining structures.

Analytical solutions based on arching theories take into account the normal and shear stresses induced at the interface between walls and fill. The magnitude of the stress transfer to abutments depends on unit weight of the fill, width and depth of the opening, interface friction angle, and reaction coefficient of the fill $K$, which in turn depends on material property and wall displacement. The theoretical value of $K$ can vary from a fully passive ($K = K_p$) to a fully active ($K = K_a$) conditions when the wall moves inward or outward respectively. The value of $K$ may thus vary within a wide range of values, typically from about 0.3 to 3 and more. Relatively large strains (above 1 to 5%) may however be required for the full lateral pressure to develop (e.g., Das 1999).
3 ANALYTICAL METHOD

Analytical solutions can provide valuable information to engineers faced with geomechanical problems. Such closed-form expressions are particularly useful for the preliminary evaluation of particular loading conditions, and also to validate some numerical simulations for well defined (but often simplified) situations.

In the case of backfilled mined stopes, the Marston theory (Marston 1930; McCarthy 1988) provides a specific solution ensuing from the general arching concept. Initially developed for buried conduits in ditches, this solution can be used to estimate the vertical and lateral loads on the stope floor and walls. This theory takes into account the fill weight and shearing forces between the vertical walls and filling material at a given depth. Figure 1 shows a schematic view of a typical vertical narrow backfilled stope, with the different force components (based on uniformly distributed stresses on the isolated element).

In Figure 1, \( H \) is the backfill height and \( B \) is the stope width. At position \( h \), the horizontal layer element is subjected to a lateral compressive force \( C \), a shearing force \( S \), and the vertical forces \( V \) and \( V + dV \); \( W \) represents the weight of the backfill in this layer (for a unit thickness), given by:

\[
W = \gamma B \, dh
\]

where \( \gamma \) is the unit weight of the backfill, and \( dh \) is the size of the layer element.

Using the relationship that exists between the vertical stress \( \sigma_v \) and horizontal stress \( \sigma_H \) in the backfill layer element, the lateral compressive force \( C \) can be expressed as:

\[
C = dh \, \sigma_h = dh \, K \, \sigma_v = dh \, K \, \frac{V}{B}
\]

where the reaction coefficient \( K \) is the ratio of the lateral stress to vertical stress

\[
K = \frac{\sigma_h}{\sigma_v}
\]

The equilibrium of the element implies also that:

\[
V + dV + 2S = V + W
\]

The shearing force \( S \) is defined with the commonly used Coulomb criterion:

\[
S = \sigma_h \tan \phi' \, dh = K \sigma_v \tan \phi' \, dh
\]

where \( \phi' \) is the effective friction angle between the backfill and the wall; \( \phi' \) is usually taken as the internal friction angle of the fill material.

From equations (1) to (5), one obtains:

\[
dV = \gamma B \, dh - 2K \frac{V}{B} \tan \phi' \, dh
\]

By solving equation (6) in the form given by the Marston theory (McCarthy 1988), the vertical stress acting across the floor of the stope can be written as follow:

\[
\sigma_{vH} = \gamma B \left( \frac{1 - \exp(-2KH \tan \phi'/B)}{2K \tan \phi'} \right)
\]

This stress is considered to be uniformly distributed over the entire width \( B \). The corresponding horizontal stress at depth \( H \) is then given by:

\[
\sigma_{hH} = \gamma B \left( \frac{1 - \exp(-2KH \tan \phi'/B)}{2 \tan \phi'} \right)
\]

Figure 2 (a & b) shows values of \( \sigma_{vH} \) and \( \sigma_{hH} \) calculated for different backfill heights, compared to the overburden pressure (i.e., \( \sigma_{vH} = \gamma H \) and \( \sigma_{hH} = K \sigma_{vH} \)). It can be seen that the full overburden load represents the upper-bound condition, which becomes applicable for low fill thickness or for very wide stopes (when arching can not develop within the backfill). These two figures also show that for relatively narrow stopes, the load transfer can be significant and reduce both the horizontal stress on the walls and the vertical stress on the floor of the opening. This type of response is in accordance with in situ measurements reported by Knutsson (1981).

4 NUMERICAL CALCULATIONS

A series of preliminary calculations have been performed to evaluate, with a commonly used
software, the response of backfill in stopes. Modeling was conducted with PHASES2, a numerical code based on the finite element method (RocScience 2002). Many calculations have been done (Arnoldi 2002), and a few sample results are presented here.

Figure 2: Comparison between overburden stress and vertical (SIGv) and horizontal (SIGh) stresses estimated with the Marston theory (Eqs. 7 and 8), for different backfill height (a) and width (b); $\phi' = 30^\circ$, $\gamma = 0.02$ MN/m$^3$, and $K = K_0 = 0.5$.

Figure 3 shows the geometry and material properties used for the numerical calculations presented here. The relatively narrow stope dimension is 45m×6m. The natural in situ vertical stress $\sigma_v$ in the rock mass is obtained by considering the overburden weight at a depth of about 250 m. In the example shown here, the natural in situ horizontal stress $\sigma_h$ is twice the vertical stress $\sigma_v$; this corresponds to a representative natural stress ratio $\sigma_h/\sigma_v$ for the Canadian shield. The rock mass is homogeneous, isotropic and linear elastic, while the granular backfill has a elasto-plastic (or elastic-peak-residual) behavior. The rock mass obeys an updated version of the Hoek-Brown failure criterion (Hoek et al. 1995), while the backfill failure condition is described by a Coulomb criterion without cohesion. The effect of cohesion due backfill cementation is not presented here, although it has been considered in other calculations. The absence of cohesion does not affect the main trends of the results. Cohesion can however amplify the local build up of internal (mean) stresses in the backfill close to the walls (see below) because it increases the $K$ value and corresponding lateral (horizontal) stresses (e.g., Mazindrani and Ganjali 1995).

Figure 3: A vertical stope with backfill (not to scale); the main properties of the rock mass and backfill are given in the figure using classical geomechanical notations.

The interfaces between the backfill and the rock mass are treated as discontinuities with a Coulomb failure criterion (without cohesion). The friction angle $\phi'$ of the backfill is also used for the interface between the backfill and rock mass. These interfaces are represented by joint elements (included within the code), which allow relative displacement between the two materials.

Figures 4 and 5 show typical results. In Figure 5, the numerical calculations are compared to analytical results for the induced stress distribution along three profiles:

i) horizontal axis at the bottom of the stope (Fig. 5a);

ii) vertical axis along the wall (Fig. 5b);
iii) the vertical center line in the stope (Fig. 5c).

For the Marston theory, three cases have been considered for the reaction coefficient $K$ (Bowles 1988):

a) pressure at rest; in this case, the reaction coefficient is given by (Jaky 1948):

$$K = K_0 = 1 - \sin \phi' = 0.5 \quad (9)$$

b) active pressure; the reaction coefficient is expressed as:

$$K = K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = 0.33 \quad (10)$$

c) passive pressure; the reaction coefficient becomes:

$$K = K_p = \frac{1 + \sin \phi'}{1 - \sin \phi'} = 3.0 \quad (11)$$

The numerical calculations show that the vertical stress at the bottom of the stope is lower than the overburden weight of the fill, thus indicating that some form of arching occurs. It can also be seen that, for this case, the maximum value of $\sigma_v$ occurs around mid-height. At that location, the vertical stress is larger than the overburden stress. This is due, in part, to the inward convergence of the walls (up to 14 mm on both sides; see Fig. 4c) that tends to squeeze the fill and increase the internal pressure (i.e. mean stress) and corresponding horizontal stress (Fig. 4a). However, the magnitude of this effect could be overestimated because it is expected that a large part of convergence may occur before backfill is put in place. Also, the constitutive model used in the code does not consider yielding of the fill material due to high mean stress. To properly address this aspect, a different type of calculation approach would be required. In that regard, the authors are working on introducing (in another numerical code) the MSDP$_u$ criterion (Aubertin et al. 2000) with a cap yielding function under isotropic loading, to better represent this behavior.
both the numerical and overburden stresses. The magnitude of this stress largely depends on the $K$ value adopted for the analytical calculations.

In the case of an isolated narrow backfilled stopes in hard rock at relatively small depth, the wall movement is expected to be fairly small once backfill is put in place, so the actual response would be close to the at rest condition ($K = K_0$). In this case, the fully passive condition is most certainly exaggerated (i.e., the load transfer to the abutment is too large), while the fully active condition is not realistic considering the movement required to induce it.

5 DISCUSSION

When analyzing the comparative results presented above, it is useful to recall some of the inherent limitations of the analytical solutions used here. First, it assumes a uniformly distributed vertical stress along any horizontal planes, which is not in accordance with the numerical calculations (see Figs. 4 and 5). Secondly, the theory is based on a simplified use of limit equilibrium with the Coulomb failure criterion, without any reference to the actual strains (vertical, horizontal) in the fill. As mentioned above, relative wall displacement is indirectly considered through the value of $K$ ($K_a \leq K \leq K_p$), but there is some uncertainty on the actual value of this coefficient even when material properties are known. Hence, the values of $\sigma_{vH}$ and $\sigma_{hh}$ along the walls are not easily defined with this approach (as it stands now).

The analytical calculation results have also been compared with other analytical solutions, such as the one proposed by Iglesia et al. (1999) based on the trap door problem. Although not shown here (because of space limitations), the latter solution tends to give vertical stresses that are comparable to those obtained with the Marston theory for the sample problem considered here. Hence, there is again an underestimate of the load on the floor when compared to the numerical results. In addition, it must be mentioned that the Iglesia et al. (1999) solution and the Marston theory sometimes give very different results for widths that are much larger than the one shown above.

So far, only vertical stopes have been addressed. In many cases, this would not be representative of field conditions. Inclined footwalls and hanging-walls can have a non negligible effect on how loads are distributed. To illustrate...
that, an inclined stope has been analyzed with the same numerical code.

Figure 6 shows the geometry of the inclined backfilled stope. The rock mass and fill properties are the same as in the case of Figure 3. Knutsson (1981) mentions that a wall inclination of less than 30° would have a fairly minimal influence (less than 10%) on the stress magnitude (so analytical solutions could be applied for relatively small angles). An inclination angle \( \alpha \) of 45° was used here to amplify the influence of geometry. An isotropic natural stress field (\( \sigma_v = \sigma_h \)) is imposed to minimize convergence of the walls.

![Figure 6: The inclined backfilled stope (properties are given in Fig 3).](image)

As expected, the results shown in Figure 7 indicates that the load distribution is different between the vertical and inclined stopes. For the inclined stope, arching effects appear less well developed, but there is nevertheless a portion of the pressure due to fill weight that is partially transferred to the foot wall. In this case, Figure 7a shows that the vertical stress on the floor can vary between approximately 80% of the overburden on the left hand side (acute angle) to about 33% at the right hand side (obtuse angle). Thus, the stress distribution would be far from uniform in this situation.

In addition, at about mid-height, the vertical and horizontal stresses appear to reach their maximum values, which are larger than those of the overburden; the same limitations mentioned above for the constitutive model used for the fill equally applies here. These results also indicate that analytical solutions used above can not properly represent this type of geometrical configuration. 

![Figure 7: Vertical stress \( \sigma_{vH} \) calculated along the stope floor (a); comparison of numerical and overburden for the vertical stress \( \sigma_{vh} \) (b) and horizontal stress \( \sigma_{hh} \) (c) along the central axis of the stope.](image)

More work is then required to improve the available tools for the analysis of backfill-wall interaction in mined stopes. Other conditions also need to be investigated including the effect of complex stope geometries and mechanical
characteristics of the backfill. The excavation and backfill sequence should also be taken into account.

6 CONCLUSION

A complex stress distribution can be induced in and around backfilled stope. Analytical solutions based on arching theories are available to evaluate loads on the walls and floor of the opening. Using a sample case, such calculation results are shown in the paper.

These results indicate that a significant proportion of the fill weight can be transferred to the abutment. The analytical results are then compared to numerical calculations with a finite element code. This provides additional information that tends to indicate that the magnitude of load transfer to the walls may be overestimated by the analytical calculations. However, neither calculation methods adequately represent all important aspects of material behavior, including the backfill non linear response and its yielding under high mean stress. More work is thus required to properly assess these key issues.

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